1 Nuclear transverse momentum dependent distributions

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1.1 Introduction

Transverse momentum dependent distributions(TMDs) are a generalization of parton distribution functions (PDFs) that extend our knowledge of the nucleon structure by including the information on parton transverse momentum distribution inside nucleon/nucleus. The exploration of the nucleon TMDs may also shed light on the issues about spin-orbit correlations and quantum interference effects. On the other side, nuclear TMDs play the important role in studying the final/initial state multiple re-scattering effect. The main purpose of the paper is to demonstrate that leading power nuclear effect comes from the gauge link appears in the nuclear TMDs, in which the multiple re-scattering effect is encoded.

The extraction of the TMDs from the various high energy scatterings relies on TMD factorization theorem which has been established in the e^+e^- annihilation process [1] and semi-inclusive deep-inelastic (SIDIS) lepton-nucleon scattering [2]. It is not so clear whether TMD factorization still holds in the SIDIS off large nucleus target. In our recent work [3], we simply assume that TMD factorization still holds in SIDIS off large nuclei. Correspondingly, one can introduce leading power unpolarized nuclear TMD. For simplicity, we restrict our discussion in the light cone gauge ($A^+ = 0$) [4],

$$f_q^A(x, \vec{k}_\perp) = \int \frac{dy^-}{2\pi} \frac{d^2 y_\perp}{(2\pi)^2} e^{ixp^+y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \langle A \mid \bar{\psi}(0, \vec{0}_\perp) \frac{\gamma^+}{2} \mathcal{L}_\perp(0, y) \psi(y^-, \vec{y}_\perp) \mid A \rangle, \tag{1}$$

where the transverse gauge link $\mathcal{L}_\perp \equiv P \exp\left[-ig \int_{\vec{0}_\perp}^{\vec{y}_\perp} d\vec{\xi}_\perp \cdot \vec{A}_\perp(\infty, \vec{\xi}_\perp)\right]$. This gauge links is not only crucial to ensure the gauge invariance of the TMD parton distribution functions in the light-cone gauge but also lead to physical consequences such as single-spin asymmetry in SIDIS and Drell-Yan process [5, 6, 7]. For DIS off a nucleus target, it should also contain information about transverse momentum broadening of the struck quark due to multiple scattering inside the nucleus [3].

In the study of either cold or hot nuclear matter, parton transverse momentum broadening plays a crucial role in unraveling the medium properties. One important parameter that controls parton energy loss is the parton transport parameter $\hat{q}$ or transverse momentum broadening squared per unit of propagation length [8]. Therefore, calculation and measurement of the jet transport parameter is an important step toward understanding the intrinsic properties of the QCD medium. Much efforts have been devoted to the study of transverse momentum broadening in high energy collisions within different approaches [8, 9, 10, 11, 12, 13, 14, 15]. In the subsequent section, we start from the matrix element definition of the nuclear TMD and identify the gauge link as the main source of leading

nuclear effects. The broadened distribution have a Gaussian form as found in earlier studies [14] Such broadened distribution in turn will give rise to the suppressed the azimuthal asymmetry in SIDIS off nucleus. We will address this issue in the Sec.III.

1.2 Nuclear TMDs & nucleon TMDs

The effect of final state interaction that leads to the transverse momentum broadening has been encoded in the gauge link. We will isolate the nuclear dependent part from the gauge link and express the nuclear TMD as the convolution of the Gaussian form and nucleon TMD. Let us first insert a δ function into the TMD quark distribution function,

$$f_q^A(x,\vec{k}_\perp) = \int d^2\ell_\perp f_q^A(x,\vec{\ell}_\perp)\delta^{(2)}(\vec{k}_\perp - \vec{\ell}_\perp). \tag{2}$$

Using a Taylor expansion of the δ-function, $\delta^{(2)}(\vec{k}_\perp - \vec{\ell}_\perp) = e^{-\vec{\ell}_\perp\cdot\vec{\nabla}_{k_\perp}}\delta^{(2)}(\vec{k}_\perp)$, the quark transverse momentum distribution can be written as,

$$f_q^A(x,\vec{k}_\perp) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} d^2\ell_\perp e^{ixp^+y^- - i\vec{\ell}_\perp\cdot\vec{y}_\perp}\langle A \mid \bar{\psi}(0,\vec{0}_\perp)\frac{\gamma^+}{2}e^{i\vec{\partial}_{y_\perp}\cdot\vec{\nabla}_{k_\perp}}\mathcal{L}_\perp\psi(y^-,\vec{y}_\perp) \mid A\rangle\delta^{(2)}(\vec{k}_\perp) \tag{3}$$

after partial integration in the transverse coordinate $\vec{y}_\perp$. Notice that both the quark field and the transverse gauge link depend on $\vec{y}_\perp$. Completing the integration over the transverse momentum $\vec{\ell}_\perp$ in Eq. (3), one then has

$$f_q^A(x,\vec{k}_\perp) = \int \frac{dy^-}{2\pi}e^{ixp^+y^-}\langle A \mid \bar{\psi}(0)\frac{\gamma^+}{2}e^{\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}}\psi(y^-) \mid A\rangle\delta^{(2)}(\vec{k}_\perp). \tag{4}$$

where we define the transport operator $\vec{W}_\perp(y^-,\vec{y}_\perp)$ in the light-cone gauge as $\vec{W}_\perp(y^-,\vec{y}_\perp) \equiv i\vec{D}_\perp(y^-,\vec{y}_\perp) + g\int_{y^-}^{\infty} d\xi^- \vec{F}_{+\perp}(\xi^-,\vec{y}_\perp)$. To proceed further, we make simplifications under the assumption of a weakly bound nucleus. We first expand the exponential factor in Eq. (4) in power of the transport operator $\vec{W}_\perp(0)$. The expectation value of any odd power of the operator under any unpolarized nuclear state should vanish under the parity invariance. We therefore are left only with the even-power terms of the expansion. We further neglect the covariant derivative in the transport operator which has no leading nuclear enhancement. Because of the color confinement, the quark and gluon fields at the first order of expansion could either be all attached to a single nucleon or to two separate nucleons. The quark-gluon correlation function in the second case will have a nuclear enhancement of the order $R_A/r_N \sim A^{1/3}$ [10, 16]. We will only keep the matrix elements with the nuclear enhancement and also neglect the correlation between different nucleons by assuming the large nucleus as a weakly bound. The same approximation can be applied to the other higher twist correlations. After making all of these simplifications, one obtains nuclear TMD,

$$f_q^A(x,\vec{k}_\perp) = \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} f_q^N(x,\vec{\ell}_\perp)\ , \tag{5}$$

as a convolution of the nucleon TMD and a Gaussian with a width Δ_{2F} given by the total transverse momentum broadening squared,

$$\Delta_{2F} = \frac{1}{Af_q^N(x)}\int d^2k_\perp k_\perp^2\left[f_q^A(x,\vec{k}_\perp) - f_q^N(x,\vec{k}_\perp)\right] = \int d\xi_N^- \hat{q}_F(\xi_N)\ . \tag{6}$$

where the quark transport parameter $\hat{q}_F(\xi_N)$ is defined as

$$\hat{q}_F(\xi_N) = -\frac{g^2}{2N_c}\rho_N^A(\xi_N)\int\frac{d\xi^-}{2p^+}\langle N \mid F_{+\sigma}(0)F_+^\sigma(\xi^-)\mid N\rangle = \frac{2\pi^2\alpha_s}{N_c}\rho_N^A(\xi_N)[xf_N^g(x)]_{x=0}, \quad (7)$$

with $\rho_N^A(\xi_N)$ is the spatial nucleon density inside the nucleus and $f_g^N(x)$ is the gluon distribution function in a nucleon. Eq. 5 is our main result.

1.3 Nuclear dependence of azimuthal asymmetry in SIDIS

One can generalize the above approach to nuclear modification of higher twist TMD parton distributions. The case of twist-3 and twist-4 TMDs [17, 18, 19] which account for the $\cos\phi$ and $\cos 2\phi$ azimuthal asymmetries in SIDIS have been recently investigated in the Ref. [20, 21]. In this paper, we review the nuclear dependent $\cos\phi$ azimuthal asymmetry, more specifically, in the two kinematic regions: at small transverse momentum $P_{h\perp} \sim \Lambda_{QCD}$ and intermediate transverse momentum $\Lambda_{QCD} << P_{h\perp} << Q$, where $P_{h\perp}$ is the final state hadron momentum and Q is the virtual photon momentum. The central ingredient of the treatment in Ref. [20] is the relation between the nucleon twist-3 TMDs and nuclear ones. If we look at the jet production in SIDIS, the azimuthal asymmetry is solely determined by one twist-3 TMD distribution $f^\perp(x, k_\perp)$. The ratio of the asymmetry between the SIDIS off nucleon and nuclei is [20],

$$\frac{<\cos\phi>_{eA}}{<\cos\phi>_{eN}} = \frac{f_\perp^A(x,k_\perp)/f^A(x,k_\perp)}{f_\perp^N(x,k_\perp)/f^N(x,k_\perp)} \quad (8)$$

The ratio depends on how the twist-3 TMD distributions $f_\perp^A$ is enhanced/suppressed due to the stronger final state interaction taking place inside a nucleus. Following the same approach applied to the twist-2 TMD distribution, we relate the function $f_\perp^A$ to $f_\perp^N$,

$$f_\perp^A(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp \frac{(\vec{k}_\perp\cdot\vec{\ell}_\perp)}{\vec{k}_\perp^2} e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}} f_\perp^N(x,\ell_\perp) \quad (9)$$

Given the TMDs $f^N(x, k_\perp)$ and $f_\perp^N(x, k_\perp)$, one will be able to calculate the ration. To illustrate the nuclear dependence of the asymmetry qualitatively, we consider an ansatz of the Gaussian distributions in $k_\perp$ for both TMDs,

$$f^N(x,k_\perp) = \frac{1}{\pi\alpha}f_q^N(x)e^{-k_\perp^2/\alpha}, f_\perp^N(x,k_\perp) = \frac{1}{\pi\beta}f_{q\perp}^N(x)e^{-k_\perp^2/\beta}. \quad (10)$$

As shown in Fig.1, the azimuthal asymmetry is suppressed in eA SIDIS as compared to that in eN SIDIS.

Now let's turn to discuss the asymmetry at intermediate transverse momentum. The fact that TMDs are perturbative calculable when $p_\perp >> \Lambda_{QCD}$ or $k_\perp >> \Lambda_{QCD}$ allows us to reduce the theoretical uncertainty, since the twist-3 TMDs are poorly known so far. In the parton model, a few convolutions of TMD distributions and TMD fragmentation functions account the azimuthal asymmetry for hadron production in SIDIS [17, 18]. It turns out that fragmentation functions $H_1^\perp, \tilde{H}$ are power suppressed compared to $\tilde{D}_\perp, D$ at large $p_\perp$ [22, 23, 24]. Therefore, We are left with the leading power terms proportional to $f_1\tilde{D}_\perp, f_\perp D$ at intermediate transverse momentum region. In the current fragmentation region where $p_\perp$ is large, we make collinear expansion around $p_\perp = q_\perp$ in terms of the power

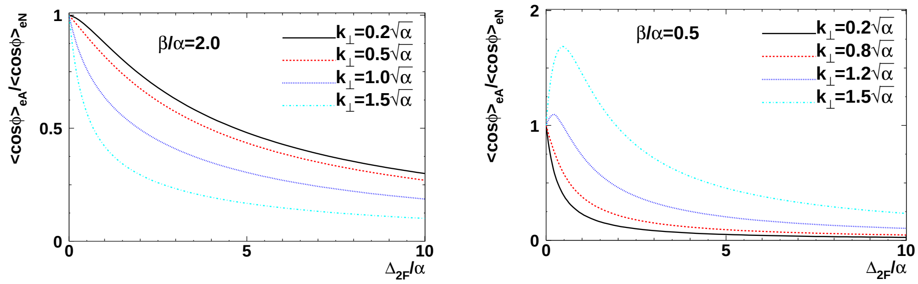

Figure 1: Ratio $\frac{\langle\cos\phi\rangle_{eA}}{\langle\cos\phi\rangle_{eN}}$ as a function of Δ_{2F} for different $k_\perp$ and the relative width β/α.

$k_\perp/q_\perp$ and keep the quadratic terms $k_\perp^2/q_\perp^2$ in order to extract the nuclear dependent contributions. After carrying out the integrals over $p_\perp$, we find the nuclear dependent azimuthal asymmetry is related to the term $D(z)\int\frac{k_\perp^2}{q_\perp^2}f_1(x,k_\perp)d^2k_\perp$. Therefore, we end up with the conclusion that the difference of the $\cos\phi_h$ azimuthal asymmetry is proportional to the transverse momentum broadening.

$$< \cos\phi_h >_{eA} - < \cos\phi_h >_{eN} \propto \int \frac{k_\perp^2}{q_\perp^2}\left[f_1^A(x,k_\perp) - f_1^N(x,k_\perp)\right] = \frac{\Delta_{2F}}{q_\perp^2} \tag{11}$$

Clearly, we can get direct handle on the crucial parameter $\hat{q}$ in describing the the properties of the QCD medium by measuring the nuclear dependent azimuthal asymmetry at intermediate transverse momentum.

1.4 Summary

We identify the gauge link appears in the nuclear TMDs as the source of leading power nuclear effects. We derive a form of nuclear transverse momentum broadening distribution by manipulating the gauge link. The broaden distribution has been expressed as the a convolution of a Gaussian distribution function and the nucleon TMD quark distribution.

Such general formalism has been extended to the twist-3 TMDs case in order to address the nuclear dependent azimuthal asymmetry in SIDIS. One finds that the $\cos\phi_h$ azimuthal asymmetry is suppressed due to the multiple re-scattering which lead to the transverse momentum broadening.

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